DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

A PLANNED COURSE FOR:

Calculus

Curriculum writing committee: Christine Marcial, Elizabeth Katz

Grade Level: 12

Date of Board Approval: ___2022_____

Calculus Grading Policy				
Marking Period	**Quiz**	**Test**	**Homework**	
MP1	150	200	35	
MP 2	160	200	40	
MP 3	160	200	40	
MP 4	160	300	40	
Total Points	630	900	155	1685
Total Percents	37.5%	53.5%	9.0%	100%

Curriculum Map

Overview:

This course is designed to develop the topics of differential and integral calculus. Emphasis is placed on limits, continuity, derivatives, and integrals of algebraic and transcendental functions of one variable. Upon completion, students should be able to select and use appropriate models, techniques, and representations for finding solutions to theoretical and applied problems with and without technology.

Students will have the opportunity to use a variety of learning methods to attain mastery of the skills and concepts necessary for success. They will demonstrate mastery through explicit textbook/eBook applications, collaboration with peers, guided inquiry, and direct instruction. Technology is integrated wherever appropriate in order to support and challenge the learning of the students.

Students need to have passed Precalculus to be enrolled in Calculus.

Time/Credit for the Course:

Full Academic Year; 1 Credit; 1 period per day

Goals:

Marking Period 1: Overview based on 45 days

Unit One Prerequisites for Calculus - 20 days

Understanding of:

- Functions and their graphs
- The library of functions
- Operations on functions
- Inverse functions
- Exponential and logarithmic functions
- Trigonometric functions

Unit Two Limits - 25 days

Understanding of:

- The definition of a limit
- Investigating limits using tables and graphs
- One-sided and two-sided limits
- Properties of limits
- Finite limits as x approaches positive and negative infinity
- Infinite limits as x approaches a constant
- The definition of continuity
- The average rate of change
- Rates of change and the derivative by definition
- Tangent and normal lines to a curve
- The Intermediate Value Theorem

Marking Period 2: Overview based on 45 days

Unit Three Derivatives - 45 days

Understanding of:

- Derivative notation
- How f '(x) might fail to exist
- How differentiability implies continuity
- Rules for differentiation on polynomials
- Rules for differentiation on $y = e^x$ and $y = \ln x$
- The Product and Quotient Rules for derivatives
- Higher-order derivatives
- Derivatives of trigonometric functions
- The Chain Rule for finding derivatives of a composite function
- Implicitly defined functions
- Related Rates in two-dimensional space

- Related Rates in three-dimensional space
- Related Rates in business applications

Marking Period 3: Overview based on 45 days

Unit Three Derivatives - 45 days

Understanding of:

- Finding absolute and relative extrema by using derivatives
- The Extreme Value Theorem
- Maximum and minimum values; critical points
- The Mean Value Theorem
- The relation of the first derivative to increasing/decreasing intervals on f(x)
- The relation of the second derivative to concavity on f(x)
- Finding inflection points and critical points
- Connecting f, f', and f" – graphically, algebraically, and analytically
- Curve sketching
- L'Hôpital's Rule
- Modeling optimization in economics, business, and industry

Marking Period 4: Overview based on 45 days

Unit Four Integrals – 45 days

Understanding of:

- Terminology and notation of antiderivatives
- Approximating the area under a curve using LRAM and RRAM
- Indefinite integration of polynomials
- Indefinite integration of quantities raised to a power
- Indefinite trigonometric integration
- Indefinite integration involving exponential and logarithmic functions
- Properties of definite integrals
- Riemann Sums for approximating the area under a curve
- The Fundamental Theorem of Calculus
- Evaluating definite integrals
- Indefinite integration using U-Substitution
- Definite integration using U-Substitution
- Determine the area of regions under a curve using integration
- Determine the area of a region between two curves using integration
- Applying definite integrals to real-world scenarios
- Volumes of solids of revolution - extension

Big Ideas:

Big Idea # 1: Numbers, measures, expressions, equations, and inequalities can represent mathematical situations and structures in many equivalent forms.

Essential Question:

- How can you extend algebraic properties and processes to linear, quadratic, absolute value, square root, piecewise, constant, identity, cubic, cube root, and reciprocal functions, and then apply them to solve real world problems?

Concept:

- Algebraic properties, processes, and representations

Competencies:

- Extend algebraic properties and processes to quadratic, exponential, and polynomial expressions and equations and apply them to solve real world problems.
- Represent exponential functions in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations; relate the growth/decay rate of the associated exponential equation to each representation.
- Represent a quadratic function in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations; relate the solution of the associated quadratic equation to each representation.

Big Idea #2: Families of functions exhibit properties and behaviors that can be recognized across representations. Functions can be transformed, combined, and composed to create new functions in mathematical and real-world situations.

Essential Question:

- How do linear, quadratic, absolute value, square root, piecewise, constant, identity, cubic, cube root, reciprocal functions, and their graphs and/or tables help us interpret events that occur in the world around us?

Concept:

- Algebraic properties, processes, and representations

Competencies:

- Extend algebraic properties and processes to quadratic, exponential, and polynomial expressions, and equations; apply them to solve real world problems.
- Represent exponential functions in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations; relate the growth/decay rate of the associated exponential equation to each representation.
- Represent a quadratic function in multiple ways, including tables, graphs, equations, and contextual situations; make connections among representations; relate the solution of the associated quadratic equation to each representation.

Big Idea #3: Mathematical functions are relationships that assign each member of one set (domain) to a unique member of another set (range), and the relationship is recognizable across representations.

Essential Question:

- How do you explain the benefits of multiple methods of representing linear, quadratic, absolute value, square root, piecewise, constant, identity, cubic, cube root, and reciprocal functions (tables, graphs, equations, and contextual situations)?

Concept:

- Algebraic properties, processes, and representations

Competencies:

- Extend algebraic properties and processes to quadratic, exponential, and polynomial expressions, and equations; apply them to solve real world problems.
- Represent exponential functions in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations; relate the growth/decay rate of the associated exponential equation to each representation.
- Represent a quadratic function in multiple ways, including tables, graphs, equations, and contextual situations; make connections among representations; relate the solution of the associated quadratic equation to each representation.

Big Idea # 4: Relations and functions are mathematical relationships that can be represented and analyzed using words, tables, graphs, and equations.

Essential Question:

- How can you extend algebraic properties and processes to quadratic, exponential and polynomial expressions and equations and apply them to solve real world problems?

Concept:

- Polynomial functions, equations, and their graphs

Competencies:

- Extend algebraic properties and processes to quadratic and polynomial expressions, equations, and their graphs; apply them to solve real world problems.
- Represent a polynomial or rational function in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations; relate the solution of the associated polynomial or rational equation to each representation.

Big Idea # 5: There are some mathematical relationships that are always true, and these relationships are used as the rules of arithmetic and algebra and are useful for writing equivalent forms of expressions and solving equations and inequalities.

Essential Question:

- How do we recognize when it is appropriate to use a derivative relationship in a situation, and what are the benefits of using this relationship?

Concept:

- algebraic properties, rules, processes, and representations

Competencies:

- Extend Algebraic properties and processes to functions and derivatives, and apply them to solve real world problems.
- Represent functions and their derivatives in multiple ways, including tables, graphs, equations, and contextual situations, and make connections among representations. Relate the derivative to each representation.

Big Idea #6: Patterns exhibit relationships that can be extended, described, and generalized.

Essential Question:

- How do you explain the benefits of multiple methods of representing functions and their derivatives & antiderivatives (tables, graphs, equations, and contextual situations)?

Concept:

- Algebraic properties, processes, and representations

Competencies:

- Represent functions in multiple ways, including tables, graphs, equations, and contextual situations; make connections among representations; relate the solution of the associated equation to each representation.

Textbook and Supplemental Resources:

Name of Textbook: Sullivan and Miranda Calculus 3rd edition

Textbook ISBN-13 #: 978-1-319-24431-6

Textbook Publisher & Year of Publication: Bedford, Freeman, & Worth ©2020

Supplemental Resources:

- BFW Sapling online eBook and student resources
- TI-84 Plus Graphing calculator
- TI-SmartView for the Smartboard
- Smart notebook gallery essentials
- Kuta Software: Calculus
- Websites such as Khan Academy and College Board

DELAWARE VALLEY SCHOOL DISTRICT

Curriculum Plan

Unit 1: Prerequisites for Calculus **Time Range in Days:** 20

Standard(s): PA Core State Standards for Mathematics
https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf

Standards Addressed:

CC.2.1.HS.F.3	CC.2.1.HS.D.1	CC.2.1.HS.D.2	CC.2.1.HS.D.3	CC.2.1.HS.D.4	CC.2.1.HS.D.7
CC.2.1.HS.D.8	CC.2.1.HS.D.9	CC.2.1.HS.D.10	CC.2.2.HS.C.1	CC.2.2.HS.C.2	CC.2.2.HS.C.4
CC.2.2.HS.C.5	CC.2.2.HS.A.1	CC.2.2.HS.A.11	CC.2.2.HS.B.1	CC.2.2.HS.B.2	CC.2.2.HS.B.3

Anchors:

- M08.B-E.1 Demonstrate an understanding of expressions and equations with radicals and integer exponents.
- M08.B-E.2 Understand the connections between proportional relationships, lines, and linear equations.
- M08.B-E.3 Analyze and solve linear equations and pairs of simultaneous linear equations.
- M08.B-F.1 Analyze and interpret functions
- M08.B-F.2 Use functions to model relationships between quantities
- A2.1.2.1 Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- A2.1.2.2 Simplify expressions involving polynomials.
- A2.2.2.2 Describe and/or determine families of functions.

Eligible Content:

- Determine whether a number is rational or irrational.
- Apply one or more properties of integer exponents to generate equivalent numerical expressions.
- Use square root and cube root symbols to represent functions.
- Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.
- Compare two different proportional relationships represented in different ways.
- Determine whether a relation is a function.
- Compare properties of two functions represented in a different ways (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions).

- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Interpret the rate of change and initial value of a function in terms of the situation it models and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
- Sketch or determine a graph that exhibits the qualitative features of a function that has been described.

Objectives:

- Evaluate a function (DOK Level Two)
- Find the difference quotient of a function (DOK Level Two)
- Find the domain of a function (DOK Level One)
- Identify the graph of a function (DOK Level Two)
- Analyze a piecewise-defined function (DOK Level Three)
- Obtain information from or about the graph of a function (DOK Level Three)
- Find the average rate of change of a function (DOK Level Two)
- Develop a library of functions (DOK Level Two and Three)
- Analyze a polynomial function and its graph (DOK Level Three and Four)
- Find the domain and the intercepts of a rational function (DOK Level Two)
- Construct a mathematical model (DOK Level Three)
- Form the sum, difference, product, and quotient of two functions (DOK Level Two)
- Form a composite function (DOK Level Two)
- Transform the graph of a function with vertical and horizontal shifts (DOK Level Two)
- Transform the graph of a function with compressions and stretches (DOK Level Two)
- Transform the graph of a function by reflecting it about the x-axis or the y-axis (DOK Level Two)
- Determine whether a function is one-to-one (DOK Level One)
- Determine the inverse of a function defined by a set of ordered pairs (DOK Level Two)
- Obtain the graph of the inverse function from the graph of a one-to-one function (DOK Level Two)
- Find the inverse of a one-to-one function defined by an equation (DOK Level Three)
- Analyze an exponential function (DOK Level Three)
- Define the number e (DOK Level One)
- Analyze a logarithmic function (DOK Level Three)
- Solve exponential equations and logarithmic equations (DOK Level Two and Three)
- Work with properties of trigonometric functions (DOK Level Two)
- Graph trigonometric functions (DOK Level Two and Three)

Core Activities and Corresponding Instructional Methods:

- Review students' prior knowledge on finding the domain of a function, identifying even and odd functions, evaluating functions and finding the average rate of change of a function.
    - Diagnostic assessment, questioning
    - Cooperative learning groups
    - Direct instruction as needed using Smart Technology
    - Guided practice using online Sapling resources

- Integrate academic and content specific vocabulary
    - Direct instruction and practice on the various parent functions (linear, quadratic, cubic, cube root, piecewise, square root, constant, identity, power, polynomial, greatest integer, reciprocal, logarithmic, absolute value, exponential, and trigonometric)
    - Classroom discussion that prompts students to compare and contrast various properties of functions using appropriate vocabulary
    - Guided practice including step-by-step written/verbal instruction using necessary vocabulary of functions (even, odd, domain, range, compression, stretch, reflection, inverse, one-to-one)
    - Utilize the graphing calculators and TI-SmartView to support key vocabulary
    - Writing activities incorporating appropriate math language

- Analyze functions and their transformations in the coordinate plane
    - Direct instruction by using visual demonstration of sets of points in the Cartesian coordinate plane by hand and in the TI-84 graphing calculator
    - Classroom discussion by using content specific vocabulary
    - Guided practice on identifying the content through Sapling resources
    - Use Graphing Utility (TI-SmartView) to enhance instruction

- Identify properties of and graph functions
    - Direct instruction and classroom discussion about properties: intervals of increase, intervals of decrease; constant intervals; local extrema and intercepts supported by visual aids on the Smart Board
    - Guided practice: Include step-by-step written/verbal explanation of the behavior of a graph
    - Cooperative group activities
    - Graphing utility practice (TI-SmartView and TI-84 graphing calculators)

- Develop students' ability to solve real world applications by applying their understanding of various functions and their properties
    - Guided practice
    - Cooperative learning groups
    - Step-by-step written/verbal explanation of the behavior of a graph
    - Graphing utility (TI-SmartView and Ti-84 graphing calculators)

Assessments:

Diagnostic:

- Teacher prepared diagnostic test
- Teacher questioning and observation

Formative:

- Teacher observations, questioning techniques
- Group activities
- Homework – example problems from the textbook and Sapling Ebook for each section
- Quizzes/graded assignments from chapter P

Summative:

- Common Assessment Chapter Exam P (Consists of both Multiple Choice and Free Response Questions)

Unit 2: Limits and Continuity

Time Range in Days: 25

Standard(s): PA Core State Standards for Mathematics
https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf

Standards Addressed:

CC.2.1.HS.F.3	CC.2.1.HS.D.1	CC.2.1.HS.D.2	CC.2.1.HS.D.3	CC.2.1.HS.D.4	CC.2.1.HS.D.7
CC.2.1.HS.D.8	CC.2.1.HS.D.9	CC.2.1.HS.D.10	CC.2.2.HS.C.1	CC.2.2.HS.C.2	CC.2.2.HS.C.4
CC.2.2.HS.C.5	CC.2.2.HS.A.1	CC.2.2.HS.A.11	CC.2.2.HS.B.1	CC.2.2.HS.B.2	CC.2.2.HS.B.3

Anchors:

- M08.B-E.3.1 Write, solve, graph, and interpret linear equations in one or two variables, using various methods.
- M08.B-F.1.1 Define, evaluate, and compare functions displayed algebraically, graphically, or numerically in tables or by verbal descriptions.
- M08.B-F.2.1 Represent or interpret functional relationships between quantities using tables, graphs, and descriptions.
- A2.2.1.1 Analyze and/or use patterns or relations.
- A2.2.2.2 Describe and/or determine families of functions.

Eligible Content:

- Graph proportional relationships, interpreting the unit rate as the slope of the graph.
- Compare two different proportional relationships represented in different ways.
- Determine whether a relation is a function.
- Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions).
- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Interpret the rate of change and initial value of a function in terms of the situation it models and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
- Sketch or determine a graph that exhibits the qualitative features of a function that has been described.

Objectives:

- Discuss the idea of a limit (DOK Level One)
- Discuss properties of limits (DOK Level Two)
- Investigate a limit using a table (DOK Level Two and Three)
- Investigate a limit using a graph (DOK Level Three)
- Define the derivative function (DOK Level One)
- Find the derivative of a function using the limit definition (DOK Level Three)
- Investigate one-sided and two-sided limits (DOK Level Three and Four)
- Find the limit of a sum, a difference, and a product (DOK Level Two and Three)
- Find the limit of a power and the limit of a root (DOK Level Two)
- Find the limit of a polynomial (DOK Level Two)
- Find the limit of a quotient (DOK Level Two)
- Find the limit of an average rate of change (DOK Level Three)
- Find the limit of a difference quotient (DOK Level Two)
- Determine whether a function is continuous at a number (DOK Level Three)
- Determine intervals on which a function is continuous (DOK Level Two)
- Use the definition and properties of continuity (DOK Level Two and Three)
- Find the limits involving trigonometric functions (DOK Level Two)
- Determine where the trigonometric functions are continuous (DOK Level Three)
- Determine where an exponential or a logarithmic function is continuous (DOK Level Three)
- Investigate limits as x approaches a constant (DOK Level Three)
- Investigate infinite limits (DOK Level Two and Three)
- Find the vertical asymptotes of a graph (DOK Level Two)
- Investigate limits at infinity (DOK Level Three and Four)
- Find the horizontal asymptotes of a graph (DOK Level Two)
- Find the asymptotes of the graph of a rational function (DOK Level Two)
- Write equations for the tangent line and the normal line to the graph of a function (DOK Level Four)
- Find the average rate of change of a function (DOK Level Three)
- Apply the Intermediate Value Theorem (DOK Level Four)

Core Activities and Corresponding Instructional Methods:

- Review students' prior knowledge on finding the domain of functions, sketching graphs of functions, and determining whether a function is continuous at a number.
    - Diagnostic assessment, questioning
    - Cooperative learning groups
    - Direct instruction as needed using Smart Technology
    - Guided practice using online Sapling resources

- Integrate academic and content specific vocabulary
    - Direct instruction and practice on the various parent functions (linear, quadratic, cubic, cube root, piecewise, square root, constant, identity, power, polynomial, greatest integer, reciprocal, logarithmic, absolute value, exponential, and trigonometric)
    - Classroom discussion that prompts students to compare various properties of functions using appropriate vocabulary
    - Guided practice including step-by-step written/verbal instruction using necessary vocabulary of functions (domain, range, asymptotes, infinite limits, limits at infinity, and continuity)
    - Utilize the graphing calculators and TI-SmartView to support key vocabulary
    - Writing activities incorporating appropriate math language

- Analyzing and finding limits analytically and graphically
    - Direct instruction by using visual demonstration of sets of points in the Cartesian coordinate plane by hand and in the TI-84 graphing calculator
    - Classroom discussion by using content specific vocabulary
    - Guided practice on identifying the content through Sapling resources
    - Use Graphing Utility (TI-SmartView) to enhance instruction

- Identify properties of continuous functions
    - Direct instruction and classroom discussion about properties: intervals of increase, intervals of decrease; constant intervals; local extrema, asymptotes and intercepts supported by visual aids on the Smart Board
    - Guided practice: Include step-by-step written/verbal explanation of the behavior of a graph
    - Cooperative group activities
    - Graphing utility practice (TI-SmartView and TI-84 graphing calculators)

- Develop students' ability to solve real world applications by applying their understanding of various functions and their properties
    - Guided practice
    - Cooperative learning groups
    - Step-by-step written/verbal explanation of the behavior of a graph
    - Graphing utility (TI-SmartView and Ti-84 graphing calculators)

Assessments:

Diagnostic:

- Teacher prepared diagnostic test, teacher questioning and observation

Formative:

- Teacher observations, questioning techniques
- Group activities
- Homework – example problems from the textbook and Sapling Ebook for each section.
- Quizzes/graded assignments from chapters 1

Summative:

- Common Assessment Chapter Exam 1 (Consists of both Multiple Choice and Free Response Questions).

DELAWARE VALLEY SCHOOL DISTRICT

Unit 3: Derivatives **Time Range in Days:** 90 days

Standard(s): PA Core State Standards for Mathematics
https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf

Standards Addressed:

CC.2.1.HS.F.3	CC.2.1.HS.D.1	CC.2.1.HS.D.2	CC.2.1.HS.D.3	CC.2.1.HS.D.4	CC.2.1.HS.D.7
CC.2.1.HS.D.8	CC.2.1.HS.D.9	CC.2.1.HS.D.10	CC.2.2.HS.C.1	CC.2.2.HS.C.2	CC.2.2.HS.C.4
CC.2.2.HS.C.5	CC.2.2.HS.A.1	CC.2.2.HS.A.11	CC.2.2.HS.B.1	CC.2.2.HS.B.2	CC.2.2.HS.B.3

Anchors:

- M08.B-E.1.1 Represent and use expressions and equations to solve problems involving radicals and integer exponents.
- M08.B-E.2.1 Analyze and describe linear relationships between two variables, using slope.
- M08.B-F.2.1 Represent or interpret functional relationships between quantities using tables, graphs, and descriptions.
- M08.C-G.2.1 Solve problems involving right triangles by applying the Pythagorean Theorem.
- M08.C-G.3.1 Apply volume formulas of cones, cylinders, and spheres.
- A2.1.2.1 Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- A2.1.2.2 Simplify expressions involving polynomials.
- A2.1.3.1 Write and/or solve non-linear equations using various methods.
- A2.1.3.2 Describe and/or determine change
- A2.2.2.1 Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.

Eligible Content:

- Graph proportional relationships, interpreting the unit rate as the slope of the graph.
- Compare two different proportional relationships represented in different ways.
- Determine whether a relation is a function.
- Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions).
- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Interpret the rate of change and initial value of a function in terms of the situation it models and in terms of its graph or a table of values.

- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
- Sketch or determine a graph that exhibits the qualitative features of a function that has been described.
- Apply formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.

Objectives:

- Find the average velocity and instantaneous velocity (DOK Level Two)
- Introduce derivative notation (DOK Level One)
- Connect differentiability and continuity (DOK Level Three)
- Find the derivative of a function at a number (DOK Level Two)
- Graph the derivative function (DOK Level Two)
- Identify where a function is not differentiable (DOK Level Three)
- Differentiate a constant function (DOK Level Two)
- Differentiate a power function (DOK Level Two)
- Differentiate the sum and difference of two functions (DOK Level Two)
- Differentiate the exponential function $y = e^x$ and $y = \ln(x)$ (DOK Level Three)
- Use the Power Rule for functions to find a derivative of a polynomial (DOK Level Three and Four)
- Differentiate the product of two functions (DOK Level Two and Three)
- Differentiate the quotient of two functions (DOK Level Two and Three)
- Find higher-order derivatives (DOK Level Two)
- Differentiate trigonometric functions: sine and cosine (DOK Level Two)
- Differentiate a composite function (DOK Level Three)
- Differentiate $y = ax$, $a>0$, $a \neq 1$ (DOK Level Two)
- Use the Chain Rule for multiple composite functions (DOK Level Three)
- Find a derivative using implicit differentiation (DOK Level Three)
- Find higher-order derivatives using implicit differentiation (DOK Level Three)
- Differentiate functions with rational exponents (DOK Level Two)
- Differentiate logarithmic functions (DOK Level Two)
- Use logarithmic differentiation (DOK Leve Three)
- Express e as a limit (DOK Level Two)
- Interpret a derivative as the slope of a tangent line to the graph of a function (DOK Level Four)
- Interpret a derivative as an instantaneous rate of change (DOK Level Four)
- Interpret a derivative as velocity or acceleration (DOK Level Four)
- Find the velocity and acceleration of an object in rectilinear motion (DOK Level Four)
- Solve Related Rates in two-dimensions (DOK Level Four)
- Solve Related Rates problems in three-dimensions (DOK Level Four)

- Solve Related Rates in business applications (DOK Level Four)
- Identify indeterminate forms of the type $\frac{0}{0}$ and $\frac{\infty}{\infty}$ (DOK Level One)
- Use L'Hôpital's Rule to find a limit (DOK Level Two and Three)
- Find the limit of an indeterminate form of the type $0\cdot\infty$, $\infty-\infty$, 0^0, 1^∞, or ∞^0 (DOK Level Two)
- Identify absolute maximum and minimum values and local extreme values of a function given its graph (DOK Level Three)
- Find critical numbers (DOK Level Two)
- Find absolute maximum and absolute minimum values using derivatives (DOK Level Two)
- Work with the Mean Value Theorem (DOK Level Four)
- Identify where a function is increasing and decreasing (DOK Level Three)
- Use the First Derivative Test to find local extrema (DOK Level Two and Three)
- Find inflection points (DOK Level Two)
- Determine the concavity of a function (DOK Level Four)
- Use the Second Derivative Test to find local extrema (DOK Level Two)
- Connect f(x), f '(x), and f "(x) graphically, algebraically, and analytically (DOK Level Four)
- Use the First and Second Derivative Tests with rectilinear motion (DOK Level Four)
- Graph a function (intervals of increase/decrease, extrema, and concavity) using calculus (DOK Level Four)
- Solve optimization problems, including those in economics, business, and industry (DOK Level Four)

Core Activities and Corresponding Instructional Methods:

- Review students' understanding of the meaning of a derivative as well as how differentiability and continuity are related
  - Diagnostic assessment, questioning
  - Cooperative learning groups
  - Direct instruction as needed using Smart Technology
  - Guided practice using online Sapling resources
- Integrate academic and content specific vocabulary
  - Direct instruction and practice on differentiation, continuity, types of discontinuity, and basic derivatives
  - Classroom discussion that prompts students to discuss differentiation, derivative rules, discontinuity, and basic antiderivatives using appropriate vocabulary.
  - Guided practice including step-by-step written/verbal instruction using necessary vocabulary of derivatives and continuity (power rule, product rule, quotient rule; removable, jump, and infinite discontinuities; inability to differentiate at a cusp, corner, and infinite)
  - Utilize the graphing calculators and TI-SmartView to support key vocabulary
  - Writing activities incorporating appropriate math language

- Analyze functions and their derivatives in terms of application
  - Direct instruction by using visual demonstration of slope of a tangent line to a curve, instantaneous rate of change, and particle motion by hand and in the TI-84 graphing calculator
  - Classroom discussion by using content specific vocabulary
  - Guided practice on identifying the content through Sapling resources
  - Use Graphing Utility (TI-SmartView) to enhance instruction
- Integrate real-world practicality of the derivative
  - Direct instruction and classroom discussion about related rates of 2-dimensional figures (circle, triangle, rectangle), related rates of 3-dimentional figures (sphere, cylinder, cone) and related rates in business (supply & demand, minimize cost)
  - Guided practice: Include step-by-step written/verbal explanation of the instantaneous rate of one variable affecting the instantaneous rate of change of another
  - Cooperative group activities
  - Graphing utility practice (TI-SmartView and TI-84 graphing calculators)
- Develop students' ability to solve real world applications by applying their understanding of derivatives, higher order derivatives, and instantaneous rate of change
  - Guided practice
  - Cooperative learning groups
  - Step-by-step written/verbal explanation of the graph of a derivative as instantaneous rate of change at any given point
  - Graphing utility (TI-SmartView and Ti-84 graphing calculators)

Assessments:

Diagnostic:

- Teacher prepared diagnostic test, teacher questioning and observation

Formative:

- Teacher observations, questioning techniques
- Group activities
- Homework – example problems from the textbook and Sapling Ebook for each section.
- Quizzes/graded assignments from chapters 2, 3, 4, and 5

Summative:

- Common Assessment Chapter Exams 2 - 5 (Consists of both Multiple Choice and Free Response Questions).

DELAWARE VALLEY SCHOOL DISTRICT

Unit 4: Integrals **Time Range in Days:** 45 days

Standard(s): PA Core State Standards for Mathematics
https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf

Standards Addressed:

CC.2.1.HS.F.3	CC.2.1.HS.D.1	CC.2.1.HS.D.2	CC.2.1.HS.D.3	CC.2.1.HS.D.4	CC.2.1.HS.D.7
CC.2.1.HS.D.8	CC.2.1.HS.D.9	CC.2.1.HS.D.10	CC.2.2.HS.C.1	CC.2.2.HS.C.2	CC.2.2.HS.C.4
CC.2.2.HS.C.5	CC.2.2.HS.A.1	CC.2.2.HS.A.11	CC.2.2.HS.B.1	CC.2.2.HS.B.2	CC.2.2.HS.B.3

Anchors:

- M08.B-E.1.1 Represent and use expressions and equations to solve problems involving radicals and integer exponents.
- M08.B-E.2.1 Analyze and describe linear relationships between two variables.
- M08.B-F.2.1 Represent or interpret functional relationships between quantities using tables, graphs, and descriptions.
- A2.1.2.1 Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- A2.1.2.2 Simplify expressions involving polynomials.
- A2.1.3.1 Write and/or solve non-linear equations using various methods.
- A2.1.3.2 Describe and/or determine change
- A2.2.2.1 Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.

Eligible Content:

- Compare two different proportional relationships represented in different ways.
- Determine whether a relation is a function.
- Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions).
- Determine the rate of change and initial value of the function from a description of a relationship or from two points, including reading from a table or graph.
- Interpret the rate of change and initial value of a function in terms of the situation it models and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch or determine a graph that exhibits the qualitative features of a function that has been described.
- Apply formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.

Objectives:

- Understand terminology and notation of antiderivatives (DOK Level One)
- Approximate an area using LRAM and RRAM (DOK Level Two)
- Find the area under the graph of a function using geometric shapes (DOK Level Two)
- Form Riemann sums (DOK Level Three and Four)
- Define a definite integral as the limit of Riemann sums (DOK Level Three)
- Approximate a definite integral using Riemann sums (DOK Level Three)
- Know conditions that guarantee a definite integral exists (DOK Level Two)
- Find a definite integral using the limit of Riemann sums (DOK Level Three)
- Form Riemann sums from a table (DOK Level Four)
- Use Part 1 of the Fundamental Theorem of Calculus (DOK Level Two)
- Use Part 2 of the Fundamental Theorem of Calculus (DOK Level Two)
- Interpret the integral as a rate of change (DOK Level Four)
- Interpret the integral as an accumulation function (DOK Level Four)
- Use properties of the definite integral (DOK Level Two and Three)
- Work with the Mean Value Theorem for integrals (DOK Level Three)
- Find the average value of a function (DOK Level One and Two)
- Interpret integrals involving rectilinear motion (DOK Level Four)
- Use properties of indefinite integrals (DOK Level Two)
- Integrate indefinite integrals using substitution (DOK Level Two)
- Use properties of definite integrals (DOK Level Two)
- Integrate definite integrals using substitution (DOK Level Two and Three)
- Find the area under a curve on a given interval (DOK Level Four)
- Find the area between the graphs of two functions on a given interval (DOK Level Four)
- Apply definite integration to real-world scenarios (DOK Level Four)
- Use the disk method to find the volume of a solid of revolution (DOK Level Three and Four)
- Use the washer method to find the volume of a solid of revolution (DOK Level Three and Four)

Core Activities and Corresponding Instructional Methods:

- Correlate integration with area under the curve using an increasing number of rectangles and their area
  - Diagnostic assessment, questioning
  - Cooperative learning groups
  - Direct instruction as needed using Smart Technology
  - Guided practice using online Sapling resources

- Integrate academic and content specific vocabulary
  - Direct instruction and practice on antidifferentiation
  - Classroom discussion that prompts students to discuss Riemann Sums, antidifferentiation, and the Fundamental Theorem of Calculus using appropriate vocabulary
  - Guided practice including step-by-step written/verbal instruction using necessary vocabulary of antiderivatives
  - Utilize the graphing calculators and TI-SmartView to support key vocabulary
  - Writing activities incorporating appropriate math language

- Analyze functions and their antiderivatives in terms of application
  - Direct instruction of the idea of integration by using visual demonstration of accumulation of areas of an increasing number of triangles by hand and in the TI-84 graphing calculator
  - Classroom discussion by using content specific vocabulary
  - Guided practice on identifying the content through Sapling resources
  - Use Graphing Utility (TI-SmartView) to enhance instruction

- Integrate real-world practicality of the antiderivative
  - Direct instruction and classroom discussion about area under a curve and between two curves on a closed interval. Contrast area under a curve to area of a regular geometric shape
  - Guided practice: Include step-by-step written/verbal explanation of the integral and rules of integration
  - Cooperative group activities
  - Graphing utility practice (TI-SmartView and TI-84 graphing calculators)

- Develop students' ability to solve real world applications by applying their understanding of antiderivatives
  - Guided practice
  - Cooperative learning groups
  - Step-by-step written/verbal explanation of the graph of a derivative as instantaneous rate of change at any given point

Assessments:

Diagnostic:

- Teacher prepared diagnostic test, teacher questioning and observation

Formative:

- Teacher observations, questioning techniques
- Group activities
- Homework – example problems from the textbook for each section.
- Quizzes/graded assignments from chapters 6 and 8

Summative:

- Common Assessment Chapter Exams 6 and 8 (Consists of both Multiple Choice and Free Response Questions).